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# APPLICATION OF A SELF-CONSISTENT NONLOCAL-HYDRODYNAMIC APPROACH TO DESCRIPTION OF THE DYNAMIC PROCESSES OF HEAT TRANSFER IN STRUCTURIZED MEDIA

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*The process of high-speed energy transfer in media with allowance for the effects of the internal structure is considered in the context of nonequilibrium statistical mechanics and the theory of nonlinear operator systems. The self-consistent nonlocal-hydrodynamic approach proposed enables one to introduce multiple scales and multiple stages of the processes of exchange of momentum and energy into the continuous description of a medium. Large-scale fluctuations transferring heat in a wave manner are generated in the medium at the initial stage of rapid heating. The measurable characteristic of these fluctuations, i.e., the dispersion of mass velocity, has been introduced into the nonequilibrium equations of state of the medium. The mechanism of regular heat conduction is established in the medium at the final stage of the process, and a synergetic formation of new structures occurs in the interval.*

**Description of High-Speed Processes.** For correct description of the processes of transfer of heat in media with a complex internal structure, one must consider the process of multistage energy exchange between different degrees of freedom and structural elements of a medium of different scale levels. The energy is transferred from a macroscopic scale level to a microscopic level of motion of individual atoms and molecules, i.e., the mechanical energy of motion of the medium's elements dissipates to a thermal form. If a medium has a complex internal structure and the scales of its elements refer to the so-called mesoscopic level (intermediate between the macro- and microlevels), before the beginning of dissipation the energy is transferred to mesoscopic degrees of freedom; it passes to increasingly smaller-scale processes until it finally reaches the microscopic scales. Thus, the mesoscopic range of the degrees of freedom acts as an energy buffer between the macro- and microscopic processes.

It has been established experimentally [1] that deformation of a medium does not immediately lead to its heating due to viscous dissipation, and the process of heat conduction (diffusion of heat) becomes steady-state at the final stage of multistage energy exchange. In the experiments on shock loading of materials, it was found that before the mechanical energy begins to dissipate to heat, large-scale fluctuations begin to be generated in the medium, where the process of formation of new internal structures subsequently begins (these structures can be referred to the mesoscopic level [2–4]).

These facts demonstrate that with high velocities and strong energy fluxes the process of multiscale energy can be nonmonotone due to the self-organization processes leading to the occurrence of new large-scale structures as a result of the collective interaction of the elements of the primary structure of a medium. It is noteworthy that, under the conditions of strong nonequilibrium, we observe structure formation even in the initially homogeneous media. For example, the occurrence of turbulence with high velocities of flow represents a synergetic process of formation of vortex structures and large-scale fluctuations [5]. A similar situation actually holds in shock loading of metals, where both shear and rotational structures frozen into the medium have been found after the passage of the wave front [2–4]. Large-scale fluctuations of the macroscopic velocity, which have been recorded inside the wave front, first grow and then decay. If velocity fluctuations decay inside the wave front, the amplitude of the wave does not decrease. This means that the process of energy exchange between the macro- and mesoscopic levels inside the front is reversible,

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and the energy from the mesolevel returns to the macrolevel almost without loss but with a certain delay. If the fluctuations do not manage to decay inside the front, part of their energy is irrevocably lost by formation of irreversible structures left behind the wave front whose amplitude decreases [6]. The quantitative characteristic of these fluctuations is the experimentally measurable dispersion of velocity under nonequilibrium conditions which acts as the temperature of mesoscopic degrees of freedom. The notions themselves of temperature, heat conduction, and heat flux away from thermodynamic equilibrium necessitate total redefinition.

Let us enumerate the basic features of high-speed processes without allowance for which we cannot describe them correctly:

(1) multiple scales of wave and dissipative structures;

(2) anisotropy related to the polarization properties of vortex systems and the asymmetry of the viscous-stress tensor and to the propagation of waves from interphase and internal boundaries;

(3) spatial order of the structures generated by both the pulsations at different points of the system and their synchronization;

(4) occurrence of a long-range or integral feedback (allowance for the influence of the internal structure of a medium on its mechanical properties) enhancing the degree of nonlinearity of the system;

(5) dependence of the effective coefficients of transfer on the regime and geometry of flow and on the distance to solid boundaries.

All these features are a direct consequence of the spatial nonlocality and memory of the structures, which in turn are generated by a substantial nonequilibrium of the processes of transfer in high-speed flows of media. However the thermodynamics of inhomogeneous media under the conditions of an arbitrary degree of nonequilibrium has not been developed at present. Hence follows all the problems related to their description [5] (we are dealing with them below).

The first problem arises even in deriving macroscopic balance equations for media with an internal structure. The most general integral balance equations which hold true without any limitations on the character of the internal structure of a medium include the boundary conditions of interaction of an open system with its surroundings. Derivation of differential balance equations is limited by the condition that the linear scale of the medium's internal structure $\lambda$ is negligible as compared to the characteristic scale of flow $L$: $\lambda << L$, $L \approx |a/\nabla a|$. Unlike the integral equations, the differential equations already do not contain boundary effects and the boundary conditions are imposed on the system irrespective of the equations.

The second problem is related to the fact that the balance equations are open and they necessitate the introduction of additional hypotheses to close them at the macroscopic level of description. Their closure within the framework of the linear thermodynamics of irreversible processes of transfer, which holds true near the local thermodynamic equilibrium, leads to classical Navier–Stokes equations.

However, with enhancement of the momentum and energy fluxes through the boundaries of the region, i.e., under nonequilibrium conditions, the interaction of the system with the boundaries gives rise to boundary layers of different kinds in which the values of the friction and the heat transfer become different from those computed from the Navier–Stokes equations. If the wall layer can be considered to be thin, the nonequilibrium effects of interaction can be included in these equations as additional terms attributed to the momentum and energy fluxes from the boundary surface:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v}\cdot\nabla)\, \mathbf{v} = -\nabla \mathbf{p} - \nabla\cdot P^{\text{N–S}} + \delta(\mathbf{r} - \mathbf{r}_b)\, \nabla\cdot(P^{\text{N–S}} - P)\,, \tag{1}$$

$$\rho \frac{\partial H}{\partial t} + \rho (\mathbf{v}\cdot\nabla)\, H = -\nabla\cdot(\mathbf{v}\cdot P^{\text{N–S}} + \mathbf{q}^{\text{N–S}}) + \delta(\mathbf{r} - \mathbf{r}_b)\, \nabla\cdot(\mathbf{v}\cdot P^{\text{N–S}} - \mathbf{v}\cdot P + \mathbf{q}^{\text{N–S}} - \mathbf{q})\,. \tag{2}$$

With increase in the degree of nonequilibrium the wall-layer thickness can increase, covering the entire flow; then the direct influence of the boundary extends deep into the region to finite distances. This means that continuous functions decaying with distance from the boundary will appear in the last terms instead of the $\delta$ functions. The occurrence of the wall layer characterized by thicknesses of the order of $O(\lambda)$ leads to difficulties in description of the system in the space of functions of a single analytical structure, including the description with the use of the apparatus of differential equations.

The next scope of problems is related to the multiple scales of the processes of transfer under nonequilibrium conditions. High-speed processes in general, just as turbulent motions, initiate a vast number of degrees of freedom at different scale levels. Ordered macroscopic motion which is described by the average field quantities stands out against the fluctuation motion. The main problems arising in averaging transfer equations involve their nonlinearity and selection of the characteristic volume. The latter difficulty is related to the introduction of the averaging scale with rapid change in the fluctuation scales and to allowance for the dependence on the symmetry properties of the volume selected.

In the general case, the momentum and energy fluxes do not coincide with the stress tensor and the vector of the heat flux in the Navier–Stokes approximation if the effects of the internal structure of the medium manifest themselves. A contribution to these fluxes, among other things, is made by the fluctuations of the mesoscopic scale level.

The role of the temperature in the nonequilibrium state is played by the spherical component $D$ of the tensor of the dispersion of the average velocity $D_{ij} = \langle v_i v_j - \langle v_i v_j \rangle \rangle$, which is attributed to the velocity fluctuations of different scale levels, including those considered to be large-scale as compared to the thermal fluctuations. The components of the tensor $D_{ij}(\mathbf{r}, t)$ can differ in directions as a consequence of the anisotropy of the nonequilibrium system. Furthermore, since the scales of mesoscopic elements can rapidly change, the mass of an element also changes. Therefore, one must introduce the mesoelement-momentum distribution instead of the velocity-distribution function [7]. Then the quasiequilibrium distribution function [8] of the medium's mesoelements by momentum takes the form

$$f_0(\mathbf{p}, \mathbf{r}, t) = (\pi\rho(\mathbf{r}, t) D)^{-3/2} \exp\left\{-\sum_{i=1}^{3} \frac{(p_i - \rho v_i(\mathbf{r}, t))^2}{\rho D(\mathbf{r}, t)}\right\}. \tag{3}$$

Expression (3) describes reversible transfer processes, which, however, can occur under nonequilibrium conditions [8]. Such a quasiequilibrium state of the system means that the functional relations of equilibrium thermodynamics hold under nonequilibrium conditions but the notions of temperature and pressure become statistical, losing their thermodynamic meaning. For example, (3) determines the nonequilibrium equation of state generalizing the notion of hydrostatic pressure to high-speed processes:

$$p = R_{\mathrm{f}}\, \rho D\,, \tag{4}$$

where $R_{\mathrm{f}} D \rightarrow RT$.

When the hydrostatic pressure has not yet formed in the medium, fluctuation pressure caused by the large-scale fluctuations of the mass velocity appears. In the case of the presence of just thermal fluctuations, which takes place near thermodynamic equilibrium, (4) takes an equilibrium form analogous to the equation of state of an ideal gas.

The employment of the first law of thermodynamics enables us to derive the balance equation for the average entropy density $s$:

$$\rho\left(\frac{\partial s}{\partial t} + (\mathbf{v}\cdot\nabla)\, s\right) + \nabla\cdot\mathbf{J}_s = \sigma\,, \tag{5}$$

where the average density of the entropy flux is $\mathbf{J}_s = \frac{\mathbf{J}_E}{\sqrt{D}}$, and the density of the entropy source, i.e., production of entropy in the system, is nonnegative according to the second law of thermodynamics $\sigma = \sum_k J_k X_k \geq 0$, where $J_k$ are the average densities of thermodynamic sources and $X_k$ are the conjugate thermodynamic forces. It is noteworthy that the solution of Eq. (5) under nonequilibrium conditions does not satisfy the Boltzmann determination of the entropy in terms of the distribution function $s = -k\int f \ln f dp$.

**Self-Consistent Nonlocal-Hydrodynamic Approach to Description of Nonequilibrium Processes of Transfer.** Continuum mechanics is, in principle, inapplicable to description of nonequilibrium processes in inhomogeneous media, and it is impossible to prescribe a rapidly changing interaction between structural elements of different scale levels in kinetic description. Therefore, we have developed a new self-consistent nonlocal-hydrodynamic approach on the basis of nonequilibrium statistical mechanics and the mechanics of resonance systems; this approach enables one to determine in dynamics the size and type of structures in the entire range from kinetic scales to macroscopic scales and their influence on the hydrodynamic characteristics of the process [5–7, 9–13]. Shortage of information on the interaction of the structural elements of a medium is compensated for with the integral information on the entire system. The integro-differential and functional equations obtained represent a system with feedback which allows control in terms of the integral and boundary conditions. Taking successive account of the dynamics of development of spatial correlations between the structural elements of the medium enables one to correctly investigate the occurrence of structural transitions in an open system.

In nonequilibrium statistical mechanics [8, 14], it has been shown that macroscopic balance equations are not completely localized under the conditions of substantial nonequilibrium. Nonlocal hydrodynamic equations with memory are obtained from the first principles [8, 14]. The integral terms in these equations are determined by the spatially nonlocal and delayed relations between the dissipative fluxes $J$ and the thermodynamic forces $X$ (gradients of hydrodynamic densities):

$$J(\mathbf{r}, t) = \int_{-\infty}^{t} dt' \int_{V} d\mathbf{r}' \,\Re\,(\mathbf{r}, \mathbf{r}', t, t')\, X\,(\mathbf{r}', t')\,. \qquad (6)$$

The relaxation kernels of transfer $\Re(\mathbf{r}, \mathbf{r}', t, t')$ which generalize the transfer coefficients to the conditions of substantial nonequilibrium represent in the general case the unknown functionals of hydrodynamic densities and their gradients.

Based on the nonlocal and delayed hydrodynamic equations, we have constructed self-consistent models of relaxation kernels with parameters that are related to the hydrodynamic quantities sought using the integral and boundary conditions [5–7, 9–13].

The self-consistent nonlocal formulation of the boundary-value problems by standard methods can be reduced to a nonlinear operator system of special form characteristic of a wide scope of resonance problems of mechanics [15, 16]. The apparatus of self-consistent nonlocal equations describes the evolution of open nonequilibrium systems actively interacting with the surroundings.

The approach developed as a universal, efficient, and economical tool of theoretical investigation of nonequilibrium processes of transfer in real media was applied to solution of a number of dynamic problems where the mechanism of heat conduction has not yet become steady-state.

**Nonstationary Heating of a Medium near a Plane Boundary.** Let us consider the problem of nonstationary heating of a medium near a plane surface of another medium having temperature $T_w(t)$. The heat flux on the surface $\mathbf{q}_w(t)$ lying in the plane $y = 0$ is directed normally to the surface.

By analogy with the thermodynamic equation relating the internal energy and the temperature in the state of local thermodynamic equilibrium, we determine the density of the internal energy of the medium $E$ in terms of the dispersion of the macroscopic velocity $D$ which acts as the temperature in nonequilibrium processes. Then we have $E = c_f D$, where the energy capacity of fluctuations excited by the nonstationary transfer of heat in the medium is $c_f = \partial E/\partial D$.

Within the framework of the self-consistent nonlocal-hydrodynamic approach, the equation of balance of the internal energy of the medium is written as follows [5, 7, 9]:

$$\frac{\partial c_f D}{\partial t} = \kappa\,(1+\alpha)\,\frac{\partial}{\partial y}\int_0^{\infty}\frac{dy}{\varepsilon}\exp\left\{-\frac{\pi}{\varepsilon^2}\,(y' - y - \gamma)\right\}\frac{\partial D}{\partial y}\,(y', t)\,. \qquad (7)$$

As the component of the kernel of transfer of the internal energy along the $y$ axis normal to the surface $\Re_y(y, y', t)$, we have employed the model expression allowing just for the nonlocal correlations along the normal to the plate and containing the following parameters: $\alpha$, correction to the effective relative thermal conductivity of the medium due to correlations and to the large-scale fluctuations generated by them; $\gamma$, polymerization shift due to the interaction of large-scale structural formations in the medium with a solid boundary; $\varepsilon$, average radius of spatial correlations.

Under nearly equilibrium conditions, the process of heat conduction is purely diffusion and it is described by the parabolic heat-conduction equation obtained from Eq. (7) in the limit $\varepsilon \to 0$, when there are no nonlocal correlations in the medium. The nonlocal kernel becomes the $\delta$ function $(1/\varepsilon)\exp\left\{-(\pi/\varepsilon^2)(y' - y - \gamma)^2\right\} \to \delta(y' - y)$, while the equation of balance of the internal energy takes the known form

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} + T_{\mathrm{w}}(t)\,(1 - \theta(y))\,. \tag{8}$$

In the limit of frozen relaxation $\varepsilon \to \infty$, i.e., on comparatively short characteristic times, one must take into account the finiteness of the velocity of propagation of disturbances in the medium $C$. Then the integral $\int_0^\infty dy = \int_0^{Ct} dy + \int_{Ct}^\infty dy = \int_0^{Ct} dy + 0$ in Eq. (8) will become the integral with a variable upper limit, $\exp\left\{-(\pi/\varepsilon^2)(y' - y - \gamma)^2\right\} \to 1$, and (7) differentiated with respect to time changes to a wave equation describing the nondissipative mechanism of transfer of the energy of large-scale fluctuations of the velocity of the medium's elements at the mesoscopic scale level (interacting with each other as elastic bodies) [6, 17]:

$$\frac{\partial^2 D}{\partial t^2} = \frac{\kappa C}{\varepsilon}\frac{\partial^2 D}{\partial y^2}\,, \quad \frac{\kappa C}{\varepsilon} = \mathrm{Pr}\, c_p C^2\,, \quad \kappa = \mathrm{Pr} c_p\, \mu\,, \quad \mu = C^2 t_{\mathrm{r}}\,, \quad \varepsilon = C t_{\mathrm{r}}\,. \tag{9}$$

According to (9), in the limit of frozen relaxation $t_{\mathrm{r}} \to \infty$, the energy is propagating with a velocity proportional to the velocity of elastic disturbances $C$ in the form of a wave of large-scale fluctuations and it does not dissipate to heat. This means that such a process is reversible.

Under the conditions of substantial nonequilibrium, when one cannot disregard spatial correlations with a finite radius $\varepsilon$, the process of heat conduction begins to form but it still preserves its wave properties. In particular, in the case of a simple exponential kernel and with allowance for the finiteness of the velocity of propagation of disturbances, Eq. (7) yields a hyperbolic-type telegraph equation of heat conduction. The proportions between two types of energy transfer depend on the scale of nonlocal correlations: the larger the quantity $\varepsilon$, the higher the fraction of the wave process in transfer of heat. It is clear that wave properties corresponding to the large degree of correlation of the behavior of the medium must prevail in the pulsed regime, and prolonged heat exchange leads to the establishment of quasistationary regimes, predominantly with a diffusion transfer of heat. In the case of strong external energy fluxes, the space-time correlations initiate a synergetic process of formation of the internal structure of the medium at mesoscale levels. The process of energy exchange at the mesoscopic level is multistaged in time, and dissipation of mechanical energy to heat is only the final stage. Therefore, it makes sense to speak of heat exchange as such only when all the mesoscopic processes related to structure formation manage to relax and the state of the medium approaches a locally equilibrium state. If velocity fluctuations are classified by scale and thermal fluctuations are described by the temperature while large-scale fluctuations are described by the velocity dispersion, Eq. (7) will become the nonlocal equation for the nonequilibrium temperature with a volumetric source caused by the disbalance of the large-scale fluctuations:

$$\frac{\partial T}{\partial t} = \kappa\,(1 + \alpha)\frac{\partial}{\partial y}\int_0^\infty \frac{dy'}{\varepsilon}\exp\left\{-\frac{\pi}{\varepsilon^2}(y' - y - \gamma)\right\}\frac{\partial T}{\partial y'} + I(t)\,, \tag{10}$$

where

$$I(t) = -\left[\frac{\partial c_m D}{\partial t} - \kappa(1+\alpha)\frac{\partial}{\partial y}\int_0^\infty \frac{dy'}{\varepsilon}\exp\left\{-\frac{\pi}{\varepsilon^2}(y'-y-\gamma)\right\}\frac{\partial D}{\partial y'}\right].$$

If the relative energy capacity of the large-scale fluctuations is $c_m \to 0$, mesofluctuations are not excited, and the scales of the fluctuations already formed are reduced to the point of their final dissipation. For finite values of $c_m$ part of the macroscopic kinetic energy is transferred to the mesoscopic level at which the process of formation of mesoscopic fluctuations and structures begins. For the condition $c_m \to \infty$ where mesofluctuations are frozen there is no energy exchange between the macro- and microscopic levels.

If we integrate (10) over the half-space and with respect to time, we obtain the integral energy balance in the form

$$\int_0^\infty T dy \Big|_{t_0}^{t} = \int_0^t q_w(t')\,dt' + \int_0^\infty dy \int_{t_0}^t I_f(t')\,dt' . \tag{11}$$

With a prescribed change in the heat flux at the boundary and in the total heat content of the medium, relation (11) simply cannot be satisfied without introducing the influence of large-scale fluctuations on the high-speed process of heat exchange.

The parameters of the nonlocal model $\alpha(t)$, $\varepsilon(t)$, and $\gamma(t)$ are determined in a self-consistent manner by the boundary relations

$$q_w(t) \equiv q_y(0, t) = -\kappa(1+\alpha)\int_0^\infty \frac{dy'}{\varepsilon}\exp\left\{-\frac{\pi}{\varepsilon^2}(y'-y)^2\right\}\frac{\partial D}{\partial y'}, \tag{12}$$

$$\dot{T}_w(t) \equiv \frac{\partial T}{\partial t}(0, t) = \kappa(1+\alpha)\int_0^\infty \frac{dy'}{\varepsilon}\frac{2\pi}{\varepsilon}(y'-\gamma)\exp\left\{-\frac{\pi}{\varepsilon^2}(y'-y)^2\right\}\frac{\partial D}{\partial y'}, \tag{13}$$

and by the integral condition (11). Since these functional relations describing the reaction of the medium to high-speed heating are substantially nonlinear, the parameters of the medium are determined by them ambiguously. Such ambiguity is an integral indication of nonequilibrium processes which are characterized by the multiple scales of dynamic structures. The nonlinear relations (11)–(13) determine the discrete spectra of scales of the medium's internal structure near the boundary and the region of a continuous spectra away from the boundary. With change in the heating conditions the discrete spectrum can change in a threshold manner with time, forming new scales of the structures, and the continuous-spectrum region can either approach the boundary or depart from it. In the limit situations where $\varepsilon \to 0$ and $\varepsilon \to \infty$, the continuous-spectrum region covers the entire half-space, pointing to the fact that the heating conditions correspond to either slow diffusion heat transfer or, conversely, purely wave nondissipative transfer.

The self-consistent formulation of the problem, i.e., the equation of transfer of the internal energy (7) and three functional relations (11)–(13) for three parameters of the model $\alpha(t)$, $\varepsilon(t)$, and $\gamma(t)$ determining the properties of the internal structure of the medium (effective thermal conductivity and structural scales), enables one to restore the dynamics of the medium's structure from the macroscopic reaction of the medium to external heating.

**Approximate Solution of the Problem.** Let us solve the problem formulated on evolution of the equilibrium-temperature field from Eq. (10) as a first approximation according to the iterative procedure developed for nonlinear operator systems [15, 16]. For this purpose, as the solution of zero approximation, we take the discontinuous temperature profile for a structureless medium with the gradient

$$\frac{\partial T^0}{\partial y} = -\frac{T-\Delta T}{\sqrt{\pi\kappa t}}\exp\left\{-\frac{y^2}{4\kappa t}\right\} - \Delta T(t)\,\delta(y) . \tag{14}$$

Substituting (14) under the integrals in expressions (12) and (13), we obtain approximate relations, where the first term describes quasistationary heating and the second term replaces the influence of the source term in Eq. (10) and determines the role of large-scale fluctuations near the heated surface [18]:

$$q_{\mathrm{w}}(t) = \rho\kappa\,(1+\alpha)\left[\frac{T-\Delta T}{\sqrt{\pi\kappa t}}\int_0^{\infty}\frac{dy'}{\varepsilon}\exp\left\{-\frac{\pi}{\varepsilon^2}(y'-\gamma)^2-\frac{y'^2}{4\kappa t}\right\}+\frac{\Delta T(t)}{\varepsilon}\exp\left\{-\frac{\pi\gamma^2}{\varepsilon^2}\right\}\right], \tag{15}$$

$$\dot{T}_{\mathrm{w}}(t) = \kappa\,(1+\alpha)\left[\frac{T-\Delta T}{\sqrt{\pi\kappa t}}\int_0^{\infty}\frac{dy'}{\varepsilon}\frac{2\pi}{\varepsilon^2}(y'+\gamma)\exp\left\{-\frac{\pi}{\varepsilon^2}(y'-\gamma)^2-\frac{y'^2}{4\kappa t}\right\}+\frac{\Delta T(t)}{\varepsilon}\frac{2\pi\gamma}{\varepsilon^2}\exp\left\{-\frac{\pi\gamma^2}{\varepsilon^2}\right\}\right]. \tag{16}$$

For prescribed functions $q_{\mathrm{w}}(t)$ and $T_{\mathrm{w}}(t)$ and evolution of the heat content $\int_0^{\infty} Tdy\Big|_{t_0}^{t}$ or the source $\int_0^{\infty} dy\int_0^{t} I(t')dt'$ (in terms of which one can easily find $\Delta T(t)$) the nonlinear relations (11), (15), and (16) determine the spectra of the values of the parameters $\alpha(t)$, $\varepsilon(t)$, and $\gamma(t)$ describing the dynamics of the medium's internal structure. Then integration of (10) for the known parameters and prescribed $q_{\mathrm{w}}(t)$, $T_{\mathrm{w}}(t)$, and $\Delta T(t)$ describes the dynamics of temperature distribution in the medium.

The analysis of the problem with allowance for the integral balance (11) has shown that for constant values of $q_{\mathrm{w}}$ and $T_{\mathrm{w}}$ we have the quasistationary regime of heating of a structurized medium, where $\Delta T = 0$, and the parameters of the medium's structure $\alpha$, $\varepsilon$, and $\gamma$ are constant. If the heat flux on the surface is not proportional to the temperature gradient, nonlocal correlations of finite radius, which grow with deviation of the behavior of the flux from the Fourier law, occur in the medium. Then the transfer of heat acquires the character of a damped wave, which follows from the nonmonotonic behavior of the temperature profiles. Otherwise the medium behaves as structureless in heating and the profiles are described by the known solution of the problem of heat conduction in a half-space.

In rapid heating, the medium does not manage to heat up and, depending on its structure, the regime of a thin dynamic layer occurs, in which $\Delta T = T_{\mathrm{w}}$ and the parameters of the medium rapidly change with time due to the fluctuation growth, which can lead to structural transitions in the intermediate situation and to the formation of new internal scales. The anisotropic growth of fluctuations near the boundary causes convection first of mesoscopic scale and then the macroscopic transfer of mass along the normal to the surface.

It is of interest to note that, as hydrophysicists have long noticed, even such a process as seasonal variation in the temperature in large ponds (slow on the scale of human life) represents a dynamic nonequilibrium process which is not described by the heat-conduction equation even on the average. Stratification with a nonmonotonic behavior of the temperature occurs in the medium, and a large temperature dispersion has been found in regions of the largest temperature gradients. This points to the generation of large-scale velocity fluctuations which come before macroscopic convection. The total energy balance is not satisfied without allowance for these fluctuations, since the heat content of water substantially lags behind the total solar energy incident on the pond surface over the same period.

## NOTATION

$\lambda$, characteristic scale of the internal structure of the medium; $L$, characteristic scale of flow of the medium; $a$, arbitrary field quantity; $\rho$, mass density; $\mathbf{v}$, mass velocity; $v_i$ and $v_j$, projections of the velocity onto the coordinate axes; $H$, enthalpy; $P$, viscous-stress tensor; $\mathbf{q}$, vector of the heat flux; $\mathbf{r}$, radius vector of a point in the coordinate space; $t$, time; $t_0$, initial instant of time; $\delta$, Dirac function; $\theta$, step function; $D$, dispersion of the mass velocity; $f$, momentum-distribution function of the particles of the medium of mesoscopic scale; $\mathbf{p}$, momentum of a mesoparticle; $R_{\mathrm{f}}$, proportionality factor in the nonequilibrium equation of state which becomes the gas constant $R$ near the equilibrium; $c_p$, specific heat of the medium at constant pressure; $c_{\mathrm{f}}$ and $c_{\mathrm{m}}$, total and relative specific energy capacities of mesofluctuations in the medium; $s$, entropy density; $E$, density of the internal energy of the medium; $\mathbf{J}_s$ and $\mathbf{J}_E$, densities of the entropy and internal-energy fluxes; $\sigma$, entropy-production density; $J$, dissipative fluxes; $X$, thermodynamic forces; $k$, Boltzmann constant; $\Re$, relaxation kernel of transfer; $T$, thermodynamic temperature of the medium; $y$, Cartesian coordinate along the normal to the boundary surface; $\kappa$, heat capacity of the medium; $\alpha$, $\varepsilon$, and $\gamma$, internal parameters of the nonlocal model; $C$, equilibrium bulk velocity of sound in the medium; $t_{\mathrm{r}}$, characteristic relaxation time; Pr, Prandtl number; $\mu$, kinematic viscosity of the medium; $I$, source term in Eq. (10); $\Delta T$, temperature jump in the zero approximation. Superscripts: N–S, values of the field quantities in the Navier–Stokes approximation; 0, zero approximation; prime, for integration variables. Subscripts: $p$, pressure; r, relaxation; f, fluctuation; m, mesofluctuation; w, wall; b, boundary; $i$ and $j$, projections of the vectors onto the coordinate axes; 0, initial.

## REFERENCES


1. G. Ravichandran, A. J. Rosakis, J. Hodovany, and P. Rosakis, in: M. D. Furnish, N. N. Thadhani, and Y. Horie (eds.), *Proc. Int. Conf. "Shock Compression of Condensed Matter"-CP620*, Atlanta, USA, 2001. Amer. Inst. Phys. 0-7354-0068-7/02 (2002), pp. 557–562.
2. Yu. I. Mescheryakov and A. K. Divakov, *DYMAT J.*, **1**, 271–287 (1994).
3. T. A. Khantuleva and Yu. I. Mescheryakov, *Fiz. Mezomekh.*, **2**, No. 5, 5–17 (1999).
4. T. A. Khantuleva and Yu. I. Mescheryakov, *Int. J. Solids Struct.*, **36**, 3105–3129 (1999).
5. B. V. Filippov and T. A. Khantuleva, in: *Proc. 15th Session of Int. School on Models of Continuum Mechanics* [in Russian], 1–10 July 2000, St. Petersburg (2000), pp. 79–90.
6. T. A. Khantuleva, in: Y. Horie, L. Daison, and N. N. Thadhani (eds.), *High-Pressure Compression of Solids VI: Old Paradigms and New Challenges*, Ch. 6, Springer, Berlin (2002), pp. 215–254.
7. T. A. Khantuleva, *Mat. Modelir.*, **11**, No. 6, 17–24 (1999).
8. D. N. Zubarev, *Modern Methods of Statistical Theory of Irreversible Processes. Advances in Science and Technology. Ser. Modern Problems of Mathematics* [in Russian], Vol. 15, Moscow (1980), pp. 152–376.
9. B. V. Filippov and T. A. Khantuleva, *Boundary Problems of Nonlocal Hydrodynamics* [in Russian], Leningrad (1984).
10. A. A. Fursenko, T. A. Khantuleva, Yu. N. Makarov, and M. S. Ramm, *Mathematical Simulation of Transport Processes of Species in Gas Phase in Modern Sputter-Deposition Technologies of Thin Films. 1. Formulation of the Problem*, Preprint No. 1595 of the A. F. Ioffe Phys. Tech. Inst., St. Petersburg (1992).
11. T. A. Khantuleva and Yu. I. Mescheryakov, *Izv. Vyssh. Uchebn. Zaved.*, *Fizika*, Pt. 1, No. 4, 62–73 (2000).
12. T. A. Khantuleva and Yu. I. Mescheryakov, *Izv. Vyssh. Uchebn. Zaved.*, *Fizika*, Pt. 2, No. 9, 66–75 (2000).
13. T. A. Khantuleva, *J. Mater. Phys. Mech.*, No. 2, 51–62 (2000).
14. V. Ya. Rudyak, *Statistical Theory of Dissipation Processes in Gases and Liquids* [in Russian], Novosibirsk (1987).
15. S. A. Vavilov and S. V. Yuhnevich, *Nonlinear Vibration Problems*, No. 25, 276–280 (1993).
16. S. A. Vavilov, *Dokl. Ross. Akad. Nauk*, **316**, No. 1, 22–26 (1991).
17. T. A. Khantuleva, in: M. D. Furnish, N. N. Thadhani, and Y. Horie (eds.), *Proc. Int. Conf. "Shock Compression of Condensed Matter"-CP620*, Atlanta, USA, 2001. Amer. Inst. Phys. 0-7354-0068-7/02 (2002), pp. 263–266.
18. T. A. Khantuleva, *Izv. Vyssh. Uchebn. Zaved.*, *Fizika*, No. 5, 71–76 (2001).